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QUARTERMASTER RESEARCH & ENGINEERING COMMAND
U S ARMY

TECHNICAL REPORT
EP-159

APPARENT THERMAL DIFFUSIVITY OF COMPOSITE FABRIC LAYERS

QUARTERMASTER RESEARCH & ENGINEERING CENTER
ENVIRONMENTAL PROTECTION RESEARCH DIVISION

JULY 1961

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2. Mathematical analysis
3. Textiles
4. Clothing
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James J. Powers

Francis W. Botsch   Michael J. Sacco

BIOPHYSICS BRANCH

Project Reference:
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FOREWORD

This report is the second in a series on the mathematical analysis of transient thermal variation in clothing. Thermal diffusivity is the physical parameter associated with transient heat conduction and is a constant for any homogeneous material. However, a uniform during field use rarely consists of a single material, and a method is needed for determining the heat conduction across a composite system of non-homogeneous layers under non-equilibrium conditions.

This report presents a mathematical derivation of apparent thermal diffusivity for composite fabric systems consisting of different cloth layers, and demonstrates a method for experimentally determining the composite thermal diffusivity based upon the general equation of conductive heat transfer in the unsteady state.

AUSTIN HENSCHEL, Ph.D.
Chief
Environmental Protection Research Division

Approved:

DALE H. SIELING, Ph.D.
Scientific Director
QM Research & Engineering Command

MERRILL L. TRIBE
Brig. Gen., USA
Commanding
QM Research & Engineering Command

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ABSTRACT

A mathematical derivation of an apparent thermal diffusivity for a generalized composite fabric system consisting of different cloth layers is presented in order to provide a parameter for analysis of unidimensional heat flow in a clothing system. An experimental method is described for obtaining the apparent thermal diffusivity as a general solution to Fourier's Conduction Equation, using a boundary step change on a semi-infinite solid composed of dissimilar fabric layers.

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APPARENT THERMAL DIFFUSIVITY OF COMPOSITE FABRIC LAYERS

1. Introduction

In a previous report (7) the concept of thermal diffusivity was applied to fabrics in order to provide a parameter for the mathematical analysis of the thermal response of cloth to non-uniform boundary conditions. However, a uniform worn during field activities rarely consists of a single type of material, and clothing for arctic use is composed of several dissimilar clothing layers. As a result, the transient response of the clothing to thermal changes at the boundary is a function not only of time and temperature but the thermal properties of the individual layers as well. Therefore, a method is needed for combining parameters of the individual layer into similar parameters for the composite layer in order to mathematically analyze transient heat transfer by conduction through the material.

Very little can be found in the literature concerning solutions to transient heat transfer problems by conduction involving composite layers. In one of his later books, Jakob (4) included a small section on composite layers; however, he was concerned with interface temperatures between two dissimilar layers, and used the methods of finite difference to approximate his solution. Capey and McKenzie (1) worked with transient heat flow in an aircraft wing, but this was a finite composite slab which was heated on one side and cooled on the other side by convection. Chen and Jensen (2) reported on transient heat by radiation in composite material. However, no one has reported a method of combining parameters to measure unsteady-state heat conduction through a dissimilar layer system.

The purpose of this report is to show that a single parameter - the thermal diffusivity - is applicable to a series of layers. The application of this complex thermal diffusivity is needed to permit a mathematical analysis of a clothing system under transient thermal conditions.

2. Theory

The general equation which describes thermal conduction through a thick slab of material which is heated at one boundary and cooled at the other is

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \tag{1}$$

where

T = temperature
t = time
x = distance into cloth from heat source

and is generally known as the Fourier Conduction Equation in One Dimension

(3). The thermal diffusivity α is directly proportional to the thermal conductivity and inversely proportional to the density and specific heat of the conducting material. In cloth, α is defined as apparent thermal diffusivity and was shown to be a reproducible characteristic of the cloth; thus the apparent thermal diffusivity serves as a parameter for adequately expressing the material's transient heat conduction properties.

In a composite layer of material, the apparent thermal diffusivity of the resultant will depend on the thermal conductivity k, and the heat capacity ρc of the combined materials. These in turn must be determined from the individual conductivities and heat capacities of the various materials and then combined into an α for the composite layers.

If the composite cloth consists of n layers of dissimilar materials, the heat flow will be assumed unidirectional, perpendicular to the layers, as shown by arrows in Figure 1. The cross-sectional area is the same for all the materials and equal to A.

Assuming the following parameters for the various layers are:

thickness	$x_1, x_2, x_3 \cdot \cdot \cdot x_n$
thermal conductivities	$k_1, k_2, k_3 \cdot \cdot \cdot k_n$
densities	$\rho_1, \rho_2, \rho_3 \cdot \cdot \cdot \rho_n$
specific heats	$c_1, c_2, c_3 \cdot \cdot \cdot c_n$

respectively for the individual layers numbered from top to bottom.

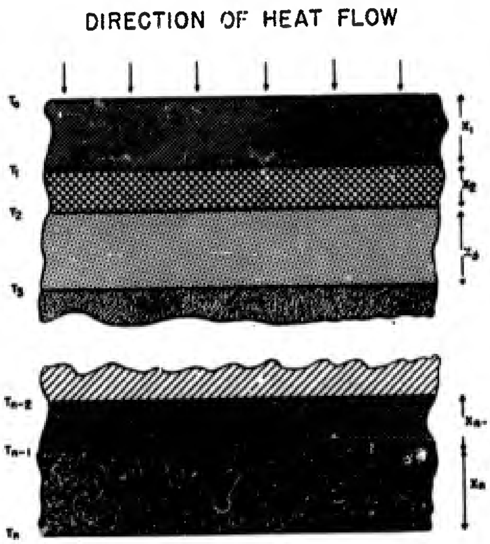

Figure 1. Cross sectional diagram of 'n' different fabric layers where arrows show direction of heat flow, T's are interface temperatures and x's are thickness of layers.

a. Thermal conductivity of the composite layer

If the temperature at the upper edge of the first material is T_o, at the interface between the first and second material it is T_1, at second interface T_2, at third interface T_3, and so on until the lower boundary which is T_n, the heat flow may be considered the same for each layer and may be represented by q. Then for material 1

$$q = \frac{k_1 A (T_o - T_1)}{x_1} \qquad (2)$$

for material 2,

$$q = \frac{k_2 A (T_1 - T_2)}{x_2} \qquad (3)$$

for material 3,

$$q = \frac{k_3 \, A(T_2 - T_3)}{x_2} \tag{4}$$

and so forth until material n, where

$$q = \frac{k_n \, A(T_{n-1} - T_n)}{x_n} \tag{5}$$

For <u>all</u> the material

$$q = \frac{k \, A(T_o - T_n)}{x_1+x_2+x_3+\cdot\cdot x_n} \tag{6}$$

where k is the thermal conductivity of the composite layer. Solving equations 2 through 5 for the temperature differences

$$T_o - T_1 = \frac{q \, x_1}{k_1 \, A} \tag{7}$$

$$T_1 - T_2 = \frac{q \, x_2}{k_2 \, A} \tag{8}$$

$$T_2 - T_3 = \frac{q \, x_3}{k_3 \, A} \tag{9}$$

and so forth until

$$T_{n-1} - T_n = \frac{q \, x_n}{k_n \, A} \tag{10}$$

Summing the n temperature differences inclusive in equation 7 through 10

$$T_o - T_n = \frac{q}{A}\left(\frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3} \cdot \cdot \cdot \frac{x_n}{k_n}\right) \tag{11}$$

Solving equation 6 for the temperature difference

$$T_o - T_n = \frac{q}{kA}(x_1 + x_2 + x_3 + \cdot \cdot \cdot x_n) \tag{12}$$

Equating equations 11 and 12

$$\frac{q}{kA}(x_1 + x_2 + x_3 + \cdot \cdot \cdot x_n) = \frac{q}{A}\left(\frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3} + \cdot \cdot \cdot \frac{x_n}{k_n}\right) \tag{13}$$

Dividing both sides by $\frac{q}{A}$

$$\frac{\sum x}{k} = \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3} + \cdot \cdot \cdot \frac{x_n}{k_n} \tag{14}$$

where

$$\sum x = x_1 + x_2 + x_3 + \cdot \cdot \cdot x_n$$

In order to simplify equation 14, both sides can be divided by $\sum x$, and making the substitutions:

$$K_1 = \frac{k_1}{x_1/\sum x}$$

$$K_2 = \frac{k_2}{x_2/\sum x} \tag{15}$$

$$K_3 = \frac{k_3}{x_3/\sum x}$$

$$\cdots\cdots$$

$$K_n = \frac{k_n}{x_n/\sum x}$$

where K_1, K_2 and so forth are the proportional part conductance of the individual layers.

Substituting 15 in 14

$$\frac{1}{k} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \cdots \frac{1}{K_n} \tag{16}$$

and

$$k = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \cdots \frac{1}{K_n}} \tag{17}$$

Simplifying equation 17

$$k = \frac{1}{\sum_{a=1}^{n} \frac{1}{K_a}} \tag{18}$$

Equation 18 gives the thermal conductivity of the composite system as a summation of the reciprocals of the proportional thermal conductances of the individual layers.

b. The heat capacity of composite layers

In a similar manner the heat capacity of the composite layer system may be obtained from the heat capacities of the individual layers.

Multiplying the volume ($A\ x_1$) by the density ρ_1, gives the mass of the

first layer, and multiplying the mass by the specific heat c_1, gives the heat stored in the layer if its temperature is increased by 1 degree. So if the temperature is increased dT in the time dt, the heat stored is

$$Q_1 = A\, x_1 \rho_1 c_1 \frac{dT}{dt} \tag{19}$$

For material 2, the heat stored when the temperature is increased dT in the time dt is

$$Q_2 = A\, x_2 \rho_2 c_2 \frac{dT}{dt} \tag{20}$$

For material 3

$$Q_3 = A\, x_3 \rho_3 c_3 \frac{dT}{dt} \tag{21}$$

and so forth until material n

$$Q_n = A\, x_n \rho_n c_n \frac{dT}{dt} \tag{22}$$

Then the total amount of heat stored Q in the composite layer is the sum of the heat stored in the individual layers. Then

$$Q = \sum_{a=1}^{n} Q_a = A(x_1 \rho_1 c_1 + x_2 \rho_2 c_2 + x_3 \rho_3 c_3 + \cdot\cdot\cdot\, x_n \rho_n c_n) \frac{dT}{dt} \tag{23}$$

The total heat stored Q in the composite layer may also be expressed as a function of the composite heat capacity for an increase of temperature dT in the time dt and

$$Q = A(x_1 + x_2 + x_3 + \cdot\cdot\cdot\, x_n) \rho\, c \frac{dT}{dt} \tag{24}$$

where ρ = density of the composite layer

and c = specific heat of the composite layer

Equating equations 23 and 24

$$A(x_1 + x_2 + \cdot\cdot\cdot\, x_n) \rho\, c \frac{dT}{dt} = A(x_1 \rho_1 c_1 + x_2 \rho_2 c_2 + \cdot\cdot\cdot\, x_n \rho_n c_n) \frac{dT}{dt} \tag{25}$$

reducing

$$(x_1 + x_2 + \cdots x_n)\,\rho c = x_1 \rho_1 c_1 + x_2 \rho_2 c_2 \cdots x_n \rho_n c_n$$

and dividing through by $\sum x$

$$\rho c = \frac{x_1 \rho_1 c_1 + x_2 \rho_2 c_2 + \cdots x_n \rho_n c_n}{\sum x}$$

Introducing a new series of constants $S_1, S_2, S_3 \ldots S_n$

such that

$$S_1 = \frac{x_1 \rho_1 c_1}{\sum x}$$

$$S_2 = \frac{x_2 \rho_2 c_2}{\sum x} \qquad (26)$$

$$S_3 = \frac{x_3 \rho_3 c_3}{\sum x}$$

$$\cdots\cdots\cdots$$

$$S_n = \frac{x_n \rho_n c_n}{\sum x}$$

Where S_1 may be defined as the proportional part of the total heat capacity of the composite system contained in the first layer.

Substituting 26 in 25

$$\rho c = S_1 + S_2 + S_3 + \cdots S_n$$

and

$$\rho c = \sum_{a=1}^{n} S_a \qquad (27)$$

Equation 27 gives the heat capacity of the composite system as a function of the proportional heat capacities of the individual layers.

c. <u>Thermal diffusivity of the composite layer</u>

The thermal parameter required as a measure of transient temperature variation by conduction is α as shown in equation 1. In cloth,

α is defined as "the apparent thermal diffusivity" and is directly proportional to the thermal conductivity and inversely proportional to the heat capacity of the conducting material. In the previous two sections, the heat capacity ρc and the thermal conductivity k were derived for a composite layer. These two parameters may now be combined into an α for a composite system and

$$\alpha = \frac{k}{\rho c} = \frac{1}{\sum_{a=1}^{n} \frac{1}{k_a} \sum_{a=1}^{n} S_a} \tag{28}$$

Equation 28 expresses the thermal diffusivity of the composite layer system as a function of the proportional thermal conductivities and specific heats of the individual layers.

In this report only two different cloths will be used in each experiment so that n = 2 in equation 28 and

$$\alpha = \frac{K_1 K_2}{(K_1 + K_2)(S_1 + S_2)} \tag{29}$$

3. Apparatus and procedure

The semi-infinite solid method, as explained in a previous report (7), was used to experimentally determine the apparent thermal diffusivity of a composite layer system. The materials comprising the semi-infinite solid were placed in a constant-temperature chamber and allowed to equilibrate. A step change in temperature was introduced at the upper plane bounding face at time t = 0 and heat began to flow into the cloth.

The step change is accomplished by bringing the upper face of the cloth into contact with the surface of a heat source which is thermo-regulated at a higher temperature. This heat source must be capable of storing a large amount of heat and delivering it, subsequent to contact with the cloth, to the interface as required to maintain it at a constant temperature.

A large cylindrical mass of iron with heater wire wound around the circumference was used as a basic heat source. The temperature of this drum heater core temperature is sensed by thermistor T_3 (Fig. 2) imbedded near the geometric center of the steel drum. The core temperature is controlled by the regulator D by adjusting the duty cycle of the circumferential heater.

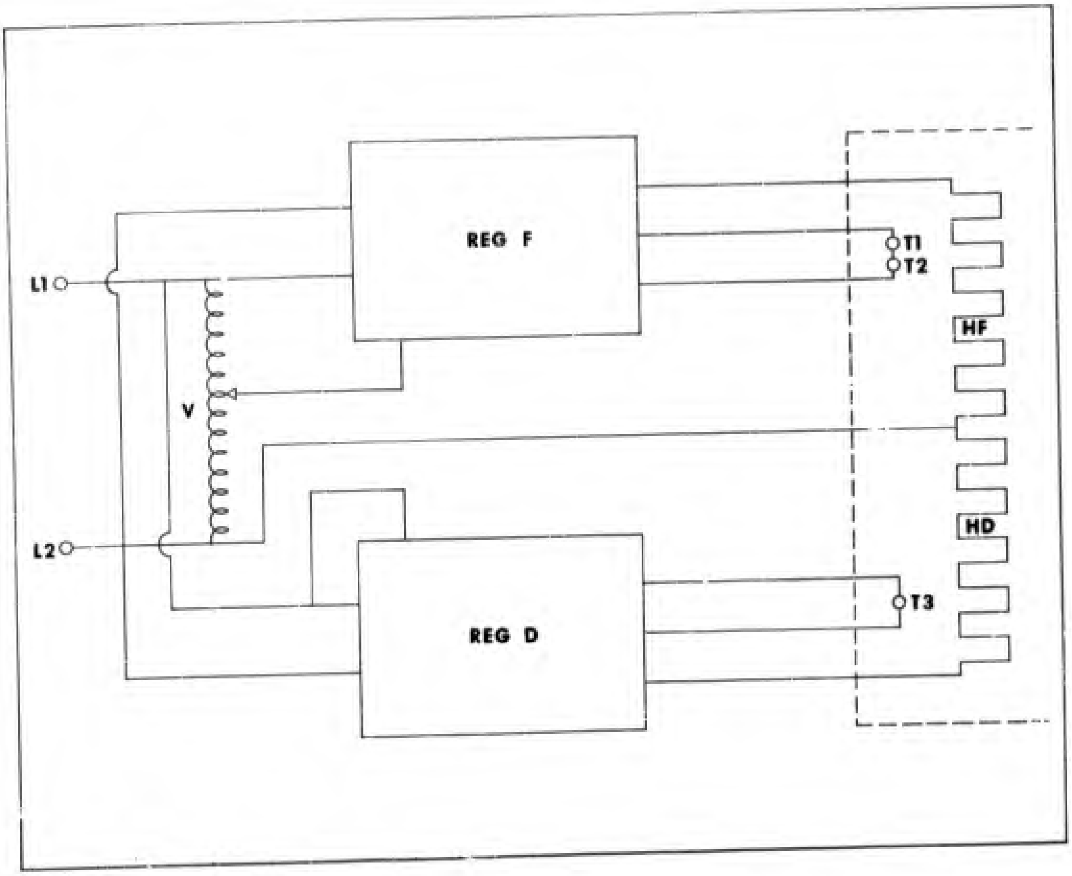

Figure 2. Wiring diagram for thermal regulation of the block and face heaters.

The drum heater is supplemented by an additional heater placed near the interface and separated from it by a thin copper plate. The copper conducts heat rapidly, thus minimizing the time lag in supplying heat to the interface (a time lag occasioned by the low thermal conductivity of the steel) and also minimizes the possibility of hot spots which could occur if the heater were placed at the surface. The use of this second heater, however, necessitates the inclusion of another heating system with associated regulatory mechanism.

The thermistor T_1 for the face is set in a hole in the copper plate close to the exposed side. A second thermistor T_2 is positioned at the junction of the steel block and copper plate. These two thermistors T_1 and T_2 are wired in series and together they maintain the face heater labeled HF in the diagram. When the cold cloth is brought in contact with the copper plate, it causes the plate to lose heat, resulting in a temperature difference between T_1 and T_2. This activates regulator F which sends heat to the heater. The influx of heat at T_2 tends to negate the transient effect at T_1, thus reducing the possibility of

overshoot of the interface temperature due to the finite time required for the diffusion of heat to the surface. Power is supplied to the face heater through variac V.

The heater and control wires L_1 and L_2 are brought up through the cylinder and attached to a terminal strip mounted on the top of the block. The top surface and the entire circumference of the block are covered by an insulating jacket of wood.

The thermocouple used for measuring the interface or boundary temperature is imbedded into but insulated from the exposed side of the copper plate. In this manner, the thermocouple makes thermal contact with the copper as well as the material to be measured.

Once the step change is realized at the cloth's upper surface, the temperature distribution within the material at some time after $t = 0$ is dependent upon the thermal properties, the initial temperature conditions and the new boundary temperature. This relationship is expressed mathematically by the particular solution of equation 1 for a semi-infinite solid (3) which is

$$T = T_s - (T_s - T_o)\left[\frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{2\sqrt{\alpha t}}} e^{-\beta^2}\, d\beta\right] \qquad (30)$$

where

α = apparent thermal diffusivity of composite layers
t = time
T_o = temperature of cloth at $t = o$
T_s = temperature of the face
T = temperature at distance x and time t
β = Gaussian error integral function of x and t
x = thickness of 1,2,3 . . . composite layers

4. Results

The first semi-infinite solid used experimentally consisted of 40 layers of cloth: 20 layers of nylon 66 alternated with 20 layers of blanket cloth. These two materials were used because all the necessary parameters were available from a previous report (7).

Thermocouples were placed in the semi-infinite solid down one composite layer (1 layer nylon plus 1 layer of blanket cloth) and two composite layers from the top, giving x's of .172 and .344 inches(that is, .437 and .874 centimeters to keep the units consistent with those

in Table 1). The cloth and chamber came into equilibrium at $T_o = 47.7^oF$, and when brought into contact with the heated cylinder resulted in a boundary temperature T_s of 110.4^oF. The resultant temperature-time curves are plotted as the circled points of Figure 3.

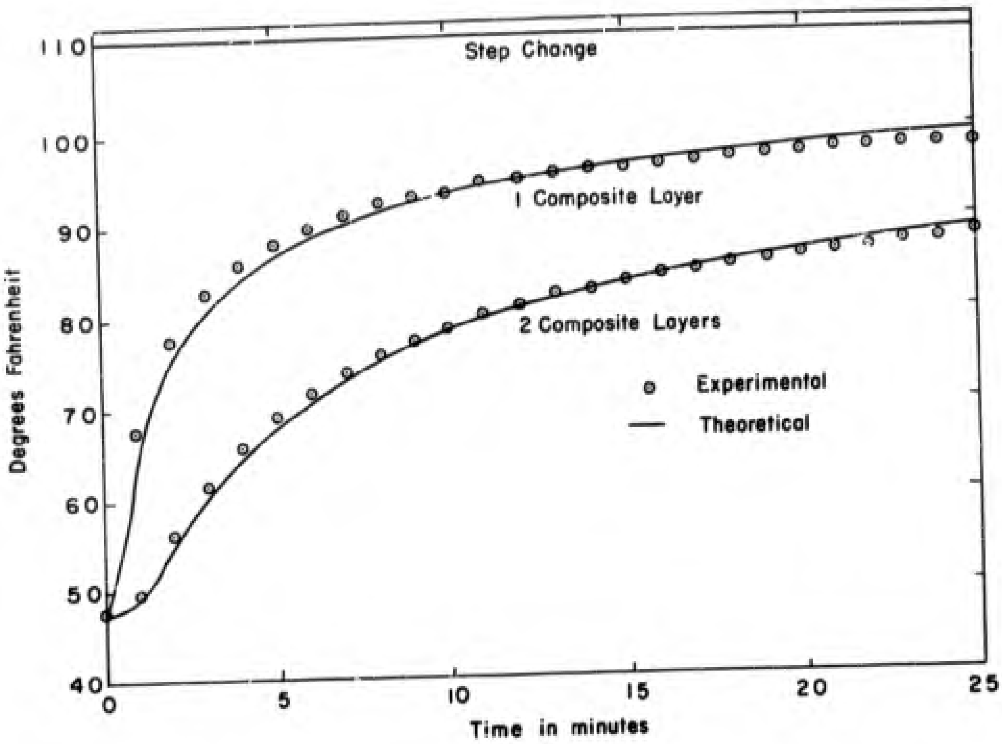

Figure 3. Transient temperature by composite layers in a semi-infinite solid of nylon and blanket cloth.

The solid curves of Figure 3 were computed using equation 30 and T_o, T_s and x's as listed above. The value of apparent thermal diffusivity used was calculated from equation 29 using the parameters of both fabrics listed in Table 1.

The computed parameters for the composite layer are also presented in Table 1 to make comparison easy. The apparent thermal diffusivity of the composite layer when converted to English units is .0120 in^2/min as compared to .0225 in^2/min for blanket cloth and .0030 in^2/min for nylon 66.

Figure 4 is a plot of the complementary semi-infinite solid. The order of the fabrics has been reversed with a layer of blanket cloth adjacent to the step change. The purpose of this was to find if any variations in the transient heat flow occur because of varying the

relative positions of the cloths. Again the circled dots are the experimental data and the solid lines are computed for an apparent thermal diffusivity of .0120 square inches per minute and with an initial temperature $T_o = 45.3$, a boundary temperature of $T_s = 102.5$, and x again equal to .172 and .344 inches.

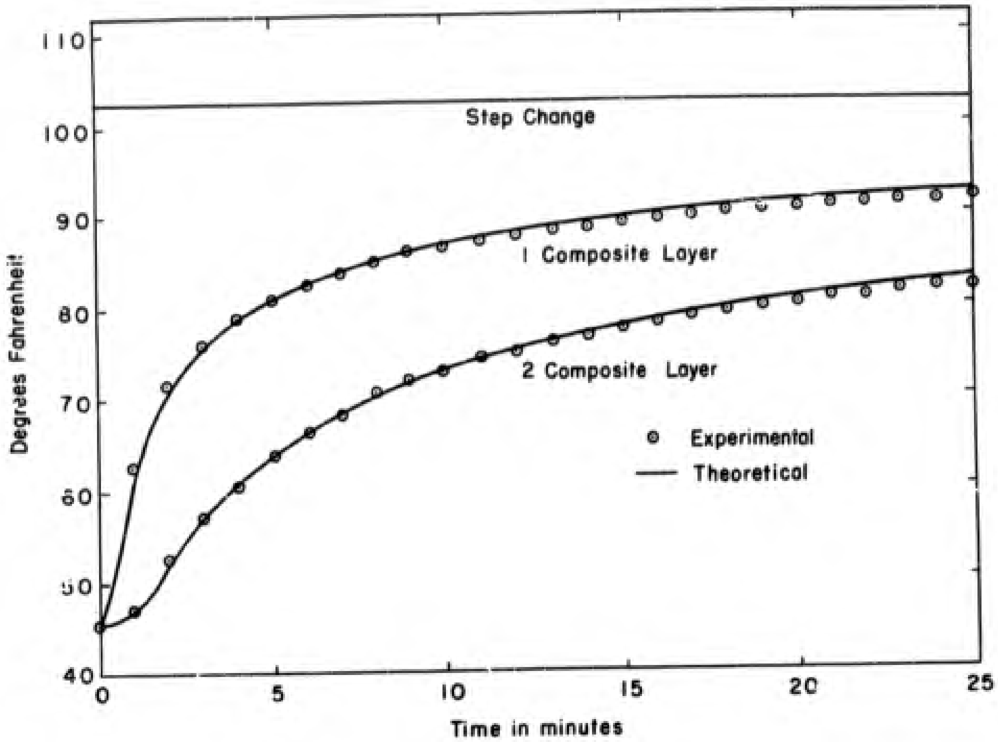

Figure 4. Transient temperature by composite layers in a semi-infinite solid of blanket cloth and nylon.

The experiment was next tried with a more practical relationship: two adjacent components of the standard arctic uniform, underwear fabric (50% wool-50% cotton, tubular knit, natural finish, 12 oz/yd^2) and shirting (85% wool-15% nylon, OG 108). The properties of these two materials and their composite layer are listed in Table II. The values of apparent thermal diffusivity in English units are: shirting .0123 in^2/min; underwear .0155 in^2/min; and the composite layer .0136 in^2/min.

The temperature-time curves for one and two composite layers are plotted in Figure 5. The solid curve was computed using equation 30 and an apparent thermal diffusivity of .0136 in^2/min with $T_o = 45^oF$, $T_s = 103^oF$ and $x = .145$ and .290 inches. The circled points are the experimental temperatures obtained with T_o, T_s and x's as stated above.

TABLE I

NECESSARY CONSTANTS FOR COMPUTING THE APPARENT THERMAL DIFFUSIVITY OF A COMPOSITE LAYER OF BLANKET CLOTH AND NYLON 66

Constants	CLOTH		
	Nylon 66	Blanket Cloth	Composite
x (cm)	.069	.368	.437
Proportional part	.158	.842	1.000
K $\left(\frac{cal}{cm/sec/^{o}C}\right)$	47.85×10^{-5}	11.08×10^{-5}	9.00×10^{-5}
S $\left(\frac{cal}{cm^3/^{o}C}\right)$	3.70×10^{-2}	3.25×10^{-2}	6.95×10^{-2}
α (cm^2/sec)	3.23×10^{-4}	24.19×10^{-4}	12.95×10^{-4}
α (in^2/min)	.0030	.0225	.0120

TABLE II

NECESSARY CONSTANTS FOR COMPUTING THE APPARENT THERMAL DIFFUSIVITY OF A COMPOSITE LAYER OF UNDERWEAR FABRIC AND NYLON-WOOL SHIRTING

Constants	CLOTH		
	Nylon-Wool Shirting	Underwear Fabric	Composite
x (cm)	.204	.165	.369
Proportional part	.553	.447	1.000
K $\left(\frac{cal}{cm/sec/^{o}C}\right)$	26.58×10^{-5}	33.78×10^{-5}	14.88×10^{-5}
S $\left(\frac{cal}{cm^3/^{o}C}\right)$	6.14×10^{-2}	4.05×10^{-2}	10.19×10^{-2}
α (cm^2/ sec)	1.33×10^{-3}	1.67×10^{-3}	1.46×10^{-3}
α (in^2/min)	.0123	.0155	.0136

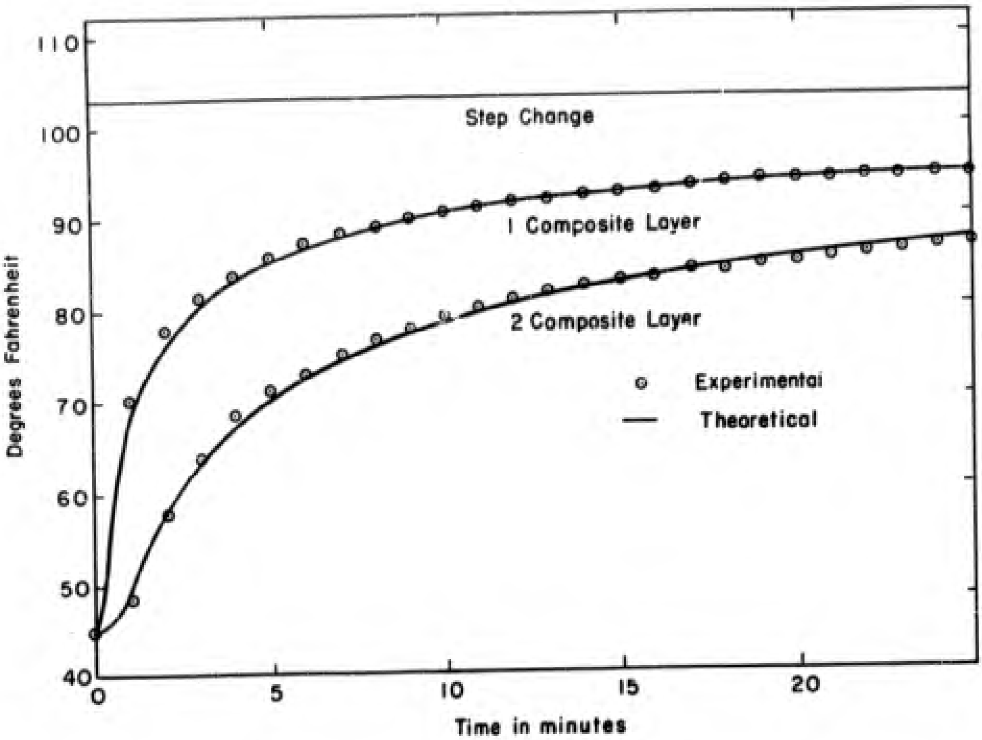

Figure 5. Transient temperature by composite layers in a semi-infinite solid of underwear cloth and nylon-wool shirting.

5. Discussion

The close agreement between the theoretical and experimental data of Figure 3, 4 and 5 shows that there is an apparent thermal diffusivity associated with a composite layer of dissimilar materials. Furthermore, this apparent thermal diffusivity can be computed for the composite layer of cloth if the thermal conductivities, thermal diffusivities and thicknesses of the individual layers are known.

The apparent thermal diffusivity of the composite system must be known in order to compute the transient heat losses through a multi-layered uniform, and it can also provide valuable insight into thermal problems in clothing design. As an example, consider the two components of the arctic uniform discussed above, the shirting and underwear fabric. The upper curve of Figure 5 is for one composite layer and shows the thermal response with time for an x of .145 inches (1 layer shirting + 1 layer underwear) and a composite apparent thermal diffusivity of .0135 in^2/min. These two constants are combined to form the upper limit to the

integral of equation 30. Inserting this upper limit into the equation as the exponent of e, it can be seen that the rate of transient response is determined by

$$-\left(\frac{x}{2\sqrt{\alpha}}\right)^2$$

For one composite layer of shirting and underwear fabric,

$$-\left(\frac{x}{2\sqrt{\alpha}}\right)^2 = -\left(\frac{.145}{2\sqrt{.0136}}\right)^2 = -(.620)^2$$

An equivalent transient response offered by shirting alone would have an upper limit equal to $-(.620)^2$ or

$$\frac{x}{2\sqrt{\alpha}} = .620$$

With an α equal to .0123 in^2/min, the thickness of shirting required to give an equal transient response can be obtained by solving for x, and x = .138 inches. Thus a layer of shirting .138 inches thick would have an equivalent transient thermal response to a composite layer .145 inches thick. Since a comparison of insulation values under steady state conditions shows that identical insulation to .145 inches of composite layer, could be equivocated by .143 inches of shirting alone, the major portion of the transient difference could not result from differences in "clo value". Although the underwear cloth may have features which make its inclusion in an arctic uniform quite desirable (such as the feel of contact with the skin and its moisture-absorbtive properties) a 5% saving in bulk and more important, a 13.7% savings in weight could be realized by using two modified layers of shirting rather than a composite layer of shirting and underwear cloth.

The example given, although not completely practical, does show that mathematical analysis of transient thermal components could lead to increased environmental protection from clothing. By properly combining materials, uniforms can be designed which have lighter weight, less bulk and greater tolerance time.

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